CHAPTER 2
Measurements and Calculations

SECTION 1
Scientific Method

SECTION 2
Units of Measurement

SECTION 3
Using Scientific Measurements

ONLINE LABS include:
- Laboratory Procedures
- Hit and Run
- The Parking Lot Collision
- Percentage of Water in Popcorn
- Accuracy and Precision in Measurements
- Specific Heat

Why It Matters Video
HMDSScience.com

Measurements and Calculations
Sometimes progress in science comes about through accidental discoveries. Most scientific advances, however, result from carefully planned investigations. The process researchers use to carry out their investigations is often called the scientific method. The scientific method is a logical approach to solving problems by observing and collecting data, formulating hypotheses, testing hypotheses, and formulating theories that are supported by data.

Main idea
Observation includes making measurements and collecting data.

Observing is the use of the senses to obtain information. Observation often involves making measurements and collecting data. The data may be descriptive (qualitative) or numerical (quantitative) in nature. Numerical information, such as the fact that a sample of copper ore has a mass of 25.7 grams, is quantitative. Non-numerical information, such as the fact that the sky is blue, is qualitative.

Experimenting involves carrying out a procedure under controlled conditions to make observations and collect data. To learn more about matter, chemists study systems. The students in Figure 1.1 are doing an experiment to test the effects absorbed water has on popcorn. A system is a specific portion of matter in a given region of space that has been selected for study during an experiment or observation. When you observe a reaction in a test tube, the test tube and its contents form a system.
Formulating Hypotheses  A graph of data can show relationships between variables. In this case, the graph shows data collected during an experiment to determine the effect of phosphorus fertilizer compounds on plant growth.

CRITICAL THINKING

Predict Outcomes  How would you finish this hypothesis:
*If* phosphorus stimulates corn-plant growth, *then*…?

MAIN IDEA

**Hypotheses are testable statements.**

As scientists examine and compare the data from their experiments, they attempt to find relationships and patterns—in other words, they make generalizations based on the data. Generalizations are statements that apply to a range of information. To make generalizations, data are sometimes organized in tables and analyzed using statistics or other mathematical techniques, often with the aid of graphs and a computer.

Scientists use generalizations about the data to formulate a **hypothesis**, or testable statement. The hypothesis serves as a basis for making predictions and for carrying out further experiments. Hypotheses are often drafted as “if-then” statements. The “then” part of the hypothesis is a prediction that is the basis for testing by experiment. Figure 1.2 shows data collected to test a hypothesis.

**Controls and Variables**

Testing a hypothesis requires experimentation that provides data to support or refute a hypothesis or theory. During testing, the experimental conditions that remain constant are called *controls*, and any condition that changes is called a *variable*. Any change observed is usually due to the effects of the variable. If testing reveals that the predictions were not correct, the hypothesis on which the predictions were based must be discarded or modified.
Main Idea

Modeling ideas helps to form theories.

When the data from experiments show that the predictions of the hypothesis are successful, scientists typically try to explain the phenomena they are studying by constructing a model. A model in science is more than a physical object; it is often an explanation of how phenomena occur and how data or events are related. Models may be visual, verbal, or mathematical.

One important model in chemistry is the atomic model of matter, which states that matter is composed of tiny particles called atoms.

If a model successfully explains many phenomena, it may become part of a theory. The atomic model is a part of the atomic theory, which you will study in the chapter “Atoms: The Building Blocks of Matter.” A theory is a broad generalization that explains a body of facts or phenomena. Theories are considered successful if they can predict the results of many new experiments. Examples of the important theories you will study in chemistry are kinetic-molecular theory and collision theory. Figure 1.3 shows where theorizing fits in the scheme of the scientific method.
Models in Chemistry

Seeing is believing, as the saying goes—but much of science deals with objects and events that cannot be seen. Models help explain the unseen, from the structures of far-off galaxies to the probable look and feel of subatomic particles. Models, however, must be based on observations. And how exactly do you model something as small as atoms or molecules?

Scientists have grappled with this problem for centuries. The atomic theory was developed by scientists, who observed the behavior of visible matter and then developed hypotheses to account for that behavior. Atoms simply seemed the most likely explanation. At first, atoms were modeled as tiny billiard balls. Then, as physicists and chemists discovered more and more about atoms, the atomic models became increasingly complex, until they included nuclei and electron clouds. Much still remains unknown. While the drawings in this book can give you some idea of what atoms and molecules look like, and they can be useful for predicting how the atoms and molecules will react under certain conditions, these models are far from perfect.

Today, however, supercomputers are creating very detailed atomic models that, although only two-dimensional, are unsurpassed in their ability to help scientists make predictions about molecular behavior in chemical reactions. Scientists feed information about the chemical behavior of a molecule into the supercomputer, which produces the model. The scientists can then manipulate the model’s orientation and the conditions under which it exists. They can even highlight certain parts of the model at different times to study different things about the molecule. In this manner, they can test hypotheses about matter and how it behaves in a way that they can see and discuss.

Computer modeling can be especially useful for studying the complex molecules found in living organisms. For example, in 2010, two scientists at the University of Houston used computers to build a three-dimensional model of an enzyme linked to both Alzheimer’s disease and cancer.

The enzyme is known as phosphoglycerate kinase, or PGK. The scientists used supercomputers to create a simulation that allowed them to change PGK’s cellular environment. One thing they found was that the enzyme was 15 times more active when the cell’s interior was crowded. Now, other researchers will probably have a better chance of figuring out how to control the enzyme so it doesn’t have such serious effects.

Yet, while these new modeling techniques promise scientists models of hitherto unknown detail, one thing is worth remembering. Like all the atomic models before them, these models are still only our best guess of the unknown using empirical evidence, i.e., evidence we can see. Also, even though the model of atoms as tiny billiard balls may have proven to be too limiting a picture, thinking of models as “right” or “wrong” misses the point of building them. Good models are not always the ones that turn out to be the “right” models. Good models are any models that help explain things we see and help us make predictions.

Questions

1. Why must all models be based on empirical observations and measurements?

2. How can scientists improve the accuracy of their computer models?
Units of Measurement

Key Terms
- quantity
- SI
- weight
- derived unit
- volume
- density
- conversion factor
- dimensional analysis

Measurements are quantitative information. A measurement is more than just a number, even in everyday life. Suppose a chef wrote a recipe, listing quantities such as 1 salt, 3 sugar, and 2 flour. Cooks could not use the recipe without more information. They would need to know whether the numbers 1, 3, and 2 represented teaspoons, tablespoons, cups, ounces, grams, or some other unit for salt, sugar, and flour, respectively.

Measurements represent quantities. A quantity is something that has magnitude, size, or amount. A quantity is not the same as a measurement. For example, the quantity represented by a teaspoon is volume. The teaspoon is a unit of measurement, while volume is a quantity. A teaspoon is a measurement standard in this country. Units of measurement compare what is to be measured with a previously defined size. Nearly every measurement is a number plus a unit. The choice of unit depends on the quantity being measured.

Many centuries ago, people sometimes marked off distances in the number of foot lengths it took to cover the distance. But this system was unsatisfactory because the number of foot lengths used to express a distance varied with the size of the measurer’s foot. Once there was agreement on a standard for foot length, confusion as to the actual length was eliminated. It no longer mattered who made the measurement, as long as the standard measuring unit was correctly applied.

**MAIN IDEA**

Scientists all over the world have agreed on a single measurement system called Le Système International d’Unités, abbreviated SI. This system was adopted in 1960 by the General Conference on Weights and Measures. SI now has seven base units, and most other units are derived from these seven. Some non-SI units are still commonly used by chemists and are also used in this book.

SI units are defined in terms of standards of measurement. The standards are objects or natural phenomena that are of constant value, easy to preserve and reproduce, and practical in size. International organizations monitor the defining process. In the United States, the National Institute of Standards and Technology (NIST) plays the main role in maintaining standards and setting style conventions. For example, numbers are written in a form that is agreed upon internationally. The number seventy-five thousand is written 75,000, not 75,000, because the comma is used in other countries to represent a decimal point.
Prefixes added to SI base units indicate larger or smaller quantities.

The seven SI base units and their standard abbreviated symbols are listed in Figure 2.1. All the other SI units can be derived from these seven fundamental units.

Prefixes added to the names of SI base units are used to represent quantities that are larger or smaller than the base units. Figure 2.2 lists SI prefixes using units of length as examples. For example, the prefix centi-, abbreviated c, represents an exponential factor of $10^{-2}$, which equals 1/100. Thus, 1 centimeter, 1 cm, equals 0.01 m, or 1/100 of a meter.

Mass

As you learned in the chapter “Matter and Change,” mass is a measure of the quantity of matter. The SI standard unit for mass is the kilogram. The standard for mass defined in Figure 2.1 is used to calibrate balances all over the world. A kilogram is about 2.2 pounds.

---

**FIGURE 2.1**

<table>
<thead>
<tr>
<th>SI BASE UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Quantity</strong></td>
</tr>
<tr>
<td>Length</td>
</tr>
<tr>
<td>Mass</td>
</tr>
<tr>
<td>Time</td>
</tr>
<tr>
<td>Temperature</td>
</tr>
<tr>
<td>Amount of substance</td>
</tr>
<tr>
<td>Electric current</td>
</tr>
<tr>
<td>Luminous intensity</td>
</tr>
</tbody>
</table>
The gram, g, which is 1/1000 of a kilogram, is more useful for measuring masses of small objects, such as flasks and beakers. One gram is about the mass of a paper clip. For even smaller objects, such as tiny quantities of chemicals, the milligram, mg, is often used. One milligram is 1/1000 of a gram, or 1/1 000,000 of a kilogram.

**Mass Versus Weight**

Mass is often confused with weight. Remember, mass is a measure of the amount of matter. **Weight** is a measure of the gravitational pull on matter. Mass is determined by comparing the mass of an object with a set of standard masses on two sides of a balance. When the masses on each are the same, the sides balance. Unlike weight, mass does not depend on gravity. Thus, weight changes as gravitational force changes, while mass does not change.

Mass is measured on instruments such as a balance, and weight is typically measured on a spring scale. Taking weight measurements involves reading the amount that an object pulls down on a spring. As the force of Earth’s gravity on an object increases, the object’s weight increases. The weight of an object on the Moon is about one-sixth of its weight on Earth.

**CHECK FOR UNDERSTANDING**

Apply The gravity on the Moon is 1/6 of Earth’s gravity. What would your weight be on the moon? Would your mass on the Moon be different from your mass on Earth? Explain.

---

**FIGURE 2.2**

**SI PREFIXES**

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Unit abbreviation</th>
<th>Exponential factor</th>
<th>Meaning</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
<td>$10^{12}$</td>
<td>1 000 000 000 000</td>
<td>1 terameter (Tm) = $1 \times 10^{12}$ m</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>$10^{9}$</td>
<td>1 000 000 000</td>
<td>1 gigameter (Gm) = $1 \times 10^{9}$ m</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>$10^{6}$</td>
<td>1 000 000</td>
<td>1 megameter (Mm) = $1 \times 10^{6}$ m</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>$10^{3}$</td>
<td>1000</td>
<td>1 kilometer (km) = 1000 m</td>
</tr>
<tr>
<td>hecto</td>
<td>h</td>
<td>$10^{2}$</td>
<td>100</td>
<td>1 hectometer (hm) = 100 m</td>
</tr>
<tr>
<td>deka</td>
<td>da</td>
<td>$10^{1}$</td>
<td>10</td>
<td>1 dekameter (dam) = 10 m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$10^{0}$</td>
<td>1</td>
<td>1 meter (m)</td>
</tr>
<tr>
<td>deci</td>
<td>d</td>
<td>$10^{-1}$</td>
<td>1/10</td>
<td>1 decimeter (dm) = 0.1 m</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>$10^{-2}$</td>
<td>1/100</td>
<td>1 centimeter (cm) = 0.01 m</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>$10^{-3}$</td>
<td>1/1000</td>
<td>1 millimeter (mm) = 0.001 m</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>$10^{-6}$</td>
<td>1/1 000 000</td>
<td>1 micrometer (µm) = $1 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>$10^{-9}$</td>
<td>1/1 000 000 000</td>
<td>1 nanometer (nm) = $1 \times 10^{-9}$ m</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>$10^{-12}$</td>
<td>1/1 000 000 000 000</td>
<td>1 picometer (pm) = $1 \times 10^{-12}$ m</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>$10^{-15}$</td>
<td>1/1 000 000 000 000 000</td>
<td>1 femtometer (fm) = $1 \times 10^{-15}$ m</td>
</tr>
<tr>
<td>atto</td>
<td>a</td>
<td>$10^{-18}$</td>
<td>1/1 000 000 000 000 000 000</td>
<td>1 attometer (am) = $1 \times 10^{-18}$ m</td>
</tr>
</tbody>
</table>
Length
The SI standard unit for length is the meter. A distance of 1 m is about the width of an average doorway. To express longer distances, the kilometer, km, is used. One kilometer equals 1000 m. To express shorter distances, the centimeter, as shown Figure 2.3, is often used. The centimeter is about the size of a paper clip. From Figure 2.2, on the previous page, you can see that one centimeter equals 1/100 of a meter.

**Main Idea**
SI base units combine to form derived units.

Many SI units are combinations of the quantities shown in Figure 2.1. **Combinations of SI base units form derived units.** Derived units are produced by multiplying or dividing standard units. For example, area, a derived unit, is length times width. If both length and width are expressed in meters, the area unit equals meters times meters, or square meters, abbreviated m². Some derived units are shown in Figure 2.4. The last column of Figure 2.4 shows the combination of fundamental units used to obtain derived units. Figure 2.5, on the next page, shows a speedometer measuring speed, another example of a derived unit.

### Derived SI Units

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Quantity symbol</th>
<th>Unit</th>
<th>Unit abbreviation</th>
<th>Derivation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area</td>
<td>$A$</td>
<td>square meter</td>
<td>m²</td>
<td>length × width</td>
</tr>
<tr>
<td>Volume</td>
<td>$V$</td>
<td>cubic meter</td>
<td>m³</td>
<td>length × width × height</td>
</tr>
<tr>
<td>Density</td>
<td>$D$</td>
<td>kilograms per cubic meter</td>
<td>kg m⁻³</td>
<td>mass / volume</td>
</tr>
<tr>
<td>Molar mass</td>
<td>$M$</td>
<td>kilograms per mole</td>
<td>kg mol⁻¹</td>
<td>mass / amount of substance</td>
</tr>
<tr>
<td>Molar volume</td>
<td>$V_m$</td>
<td>cubic meters per mole</td>
<td>m³ mol⁻¹</td>
<td>volume / amount of substance</td>
</tr>
<tr>
<td>Energy</td>
<td>$E$</td>
<td>joule</td>
<td>J</td>
<td>force × length</td>
</tr>
</tbody>
</table>
Some combination units are given their own names. For example, pressure expressed in base units is the following:

$$\text{kg/m} \cdot \text{s}^2$$

The name *pascal*, Pa, is given to this combination. You will learn more about pressure in the chapter “Gases.” Prefixes can also be added to express derived units. For example, area can be expressed in cm\(^2\), square centimeters, or mm\(^2\), square millimeters.

### Volume

**Volume** is the amount of space occupied by an object. The derived SI unit of volume is cubic meters, m\(^3\). One cubic meter is equal to the volume of a cube whose edges are 1 m long. Such a large unit is inconvenient for expressing the volume of materials in a chemistry laboratory. Instead, a smaller unit, the cubic centimeter, cm\(^3\), is often used. There are 100 centimeters in a meter, so a cubic meter contains 1 000 000 cm\(^3\).

$$1 \text{ m}^3 \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}} \times \frac{100 \text{ cm}}{1 \text{ m}} = 1 \text{,}000 \text{,}000 \text{ cm}^3$$

When chemists measure the volumes of liquids and gases, they often use a non-SI unit called the liter. The liter is equivalent to one cubic decimeter. Thus, a liter, L, is also equivalent to 1000 cm\(^3\). Another non-SI unit, the milliliter, mL, is used for smaller volumes. There are 1000 mL in 1 L. Because there are also 1000 cm\(^3\) in a liter, the two units—milliliter and cubic centimeter—are interchangeable. **Figure 2.6** shows some of these different volume measurements.

**Comparing Liquid Volumes** One liter contains 1000 mL of liquid, and 1 mL is equivalent to 1 cm\(^3\). A small perfume bottle contains about 15 mL of liquid. There are about 5 mL in 1 teaspoon. The volumetric flask (far left) and graduated cylinder (far right) are used for measuring liquid volumes in the lab.
Density

A piece of cork is lighter than a piece of lead of the same size. Liquid mercury, as shown in Figure 2.7, is heavier than water. In other words, different substances contain different masses per volume. This property is called density. Density is the ratio of mass to volume, or mass divided by volume. Density is expressed by the equation

$$D = \frac{m}{V}$$


The quantity $m$ is mass, $V$ is volume, and $D$ is density.

The SI unit for density is derived from the base units for mass and volume—the kilogram and the cubic meter, respectively—and can be expressed as kilograms per cubic meter, kg/m$^3$. This unit is inconveniently large for the density measurements you will make in the laboratory. You will often see density expressed in grams per cubic centimeter, g/cm$^3$, or grams per milliliter, g/mL. The densities of gases are generally reported either in kilograms per cubic meter, kg/m$^3$, or in grams per liter, g/L.

Density is a characteristic physical property of a substance. It does not depend on the size of a sample because as the mass of a sample increases, its volume increases proportionately. The ratio of mass to volume is constant. Therefore, density is one property that can help to identify a substance. Figure 2.8 shows the densities of some common materials. As you can see, cork has a density of only 0.24 g/cm$^3$, which is less than the density of liquid water. Because cork is less dense than water, it floats on water. Lead, on the other hand, has a density of 11.35 g/cm$^3$. The density of lead is greater than that of water, so lead sinks in water.

Note that Figure 2.8 specifies the temperatures at which the densities were measured. That is because density varies with temperature. Most objects expand as temperature increases, thereby increasing in volume. Because density is mass divided by volume, density usually decreases with increasing temperature.

### DENSITIES OF SOME FAMILIAR MATERIALS

<table>
<thead>
<tr>
<th>Solids</th>
<th>Density at 20°C (g/cm$^3$)</th>
<th>Liquids</th>
<th>Density at 20°C (g/mL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cork</td>
<td>0.24*</td>
<td>gasoline</td>
<td>0.67*</td>
</tr>
<tr>
<td>butter</td>
<td>0.86</td>
<td>ethyl alcohol</td>
<td>0.791</td>
</tr>
<tr>
<td>ice</td>
<td>0.92†</td>
<td>kerosene</td>
<td>0.82</td>
</tr>
<tr>
<td>sucrose</td>
<td>1.59</td>
<td>turpentine</td>
<td>0.87</td>
</tr>
<tr>
<td>bone</td>
<td>1.85*</td>
<td>water</td>
<td>0.998</td>
</tr>
<tr>
<td>diamond</td>
<td>3.26*</td>
<td>sea water</td>
<td>1.025**</td>
</tr>
<tr>
<td>copper</td>
<td>8.92</td>
<td>milk</td>
<td>1.031*</td>
</tr>
<tr>
<td>lead</td>
<td>11.35</td>
<td>mercury</td>
<td>13.6</td>
</tr>
<tr>
<td>† measured at 0°C</td>
<td></td>
<td>** measured at 15°C</td>
<td></td>
</tr>
</tbody>
</table>

* typical density
**PROCEDURE**

1. Using the balance, determine the mass of the 40 pennies minted prior to 1982. Repeat this measurement two more times. Average the results of the three trials to determine the average mass of the pennies.

2. Repeat step 1 with the 40 pennies minted after 1982.

3. Pour about 50 mL of water into the 100 mL graduated cylinder. Record the exact volume of the water. Add the 40 pennies minted before 1982. **CAUTION:** Add the pennies carefully so that no water is splashed out of the cylinder. Record the exact volume of the water and pennies. Repeat this process two more times. Determine the volume of the pennies for each trial. Average the results of those trials to determine the average volume of the pennies.

4. Repeat step 3 with the 40 pennies minted after 1982.

5. Review your data for any large differences between trials that could increase the error of your results. Repeat those measurements.

6. Use the average volume and average mass to calculate the average density for each group of pennies.

7. Compare the calculated average densities with the density of copper, listed in **Figure 2.8.**

**DISCUSSION**

1. Why is it best to use the results of three trials rather than a single trial for determining the density?

2. How did the densities of the two groups of pennies compare? How do you account for any difference?

3. Use the results of this investigation to formulate a hypothesis about the composition of the two groups of pennies. How could you test your hypothesis?

**MATERIALS**

- balance
- 100 mL graduated cylinder
- 40 pennies dated before 1982
- 40 pennies dated after 1982
- water

**SAFETY**

Wear safety goggles and an apron.

---

**Density**

**Sample Problem A** A sample of aluminum metal has a mass of 8.4 g. The volume of the sample is 3.1 cm$^3$. Calculate the density of aluminum.

1. **ANALYZE**
   
   Given: 
   
   - mass ($m$) = 8.4 g
   - volume ($V$) = 3.1 cm$^3$
   
   Unknown: density ($D$)

2. **PLAN**
   
   density = \( \frac{mass}{volume} \)

3. **SOLVE**
   
   density = \( \frac{8.4 \text{ g}}{3.1 \text{ cm}^3} \) = 2.7 g/cm$^3$

Continued
Main idea

Conversion factors change one unit to another.

A conversion factor is a ratio derived from the equality between two different units that can be used to convert from one unit to the other. For example, suppose you want to know how many quarters there are in a certain number of dollars. To figure out the answer, you need to know how quarters and dollars are related. There are four quarters per dollar and one dollar for every four quarters. Those facts can be expressed as ratios in four conversion factors.

\[
\frac{4 \text{ quarters}}{1 \text{ dollar}} = 1 \quad \frac{1 \text{ dollar}}{4 \text{ quarters}} = 1 \quad \frac{0.25 \text{ dollar}}{1 \text{ quarter}} = 1 \quad \frac{1 \text{ quarter}}{0.25 \text{ dollar}} = 1
\]

Notice that each conversion factor equals 1. That is because the two quantities divided in any conversion factor are equivalent to each other—as in this case, where 4 quarters equal 1 dollar. Because conversion factors are equal to 1, they can be multiplied by other factors in equations without changing the validity of the equations. You can use conversion factors to solve problems through dimensional analysis. **Dimensional analysis** is a mathematical technique that allows you to use units to solve problems involving measurements. When you want to use a conversion factor to change a unit in a problem, you can set up the problem in the following way.

\[
\text{quantity sought} = \text{quantity given} \times \text{conversion factor}
\]

For example, to determine the number of quarters in 12 dollars, you would carry out the unit conversion that allows you to change from dollars to quarters.

\[
\text{number of quarters} = 12 \text{ dollars} \times \text{conversion factor}
\]

Next, you need to decide which conversion factor gives you an answer in the desired unit. In this case, you have dollars and you want quarters. To eliminate dollars, you must divide the quantity by dollars. Therefore, the conversion factor in this case must have dollars in the denominator and quarters in the numerator: 4 quarters/1 dollar.
Thus, you would set up the calculation as follows:

\[
? \text{ quarters} = 12 \text{ dollars} \times \text{conversion factor} \\
= 12 \text{ dollars} \times \frac{4 \text{ quarters}}{1 \text{ dollar}} = 48 \text{ quarters}
\]

Notice that the dollars have divided out, leaving an answer in the desired unit—quarters.

Suppose you had guessed wrong and used \( \frac{1 \text{ dollar}}{4 \text{ quarters}} \) when choosing which of the two conversion factors to use. You would have an answer with entirely inappropriate units.

\[
? \text{ quarters} = 12 \text{ dollars} \times \frac{1 \text{ dollar}}{4 \text{ quarters}} = \frac{3 \text{ dollars}^2}{\text{quarter}}
\]

It is always best to begin with an idea of the units you will need in your final answer. When working through the Sample Problems, keep track of the units needed for the unknown quantity. Check your final answer against what you’ve written as the unknown quantity.

**Deriving Conversion Factors**

You can derive conversion factors if you know the relationship between the unit you have and the unit you want. For example, from the fact that *deci-* means “1/10,” you know that there is 1/10 of a meter per decimeter and that each meter must have 10 decimeters. Thus, from the equality \( 1 \text{ m} = 10 \text{ dm} \), you can write the following conversion factors relating meters and decimeters.

\[
\frac{1 \text{ m}}{10 \text{ dm}} \quad \text{and} \quad \frac{0.1 \text{ m}}{\text{dm}} \quad \text{and} \quad \frac{10 \text{ dm}}{\text{m}}
\]

The following sample problem illustrates an example of deriving conversion factors to make a unit conversion. In this book, when there is no digit shown in the denominator, you can assume the value is 1.

---

**Conversion Factors**

**Sample Problem B**  Express a mass of 5.712 grams in milligrams and in kilograms.

1. **ANALYZE**

   **Given:** 5.712 g
   
   **Unknown:** mass in mg and mass in kg
   
   The equality that relates grams to milligrams is
   
   \( 1 \text{ g} = 1000 \text{ mg} \)

2. **PLAN**

   The possible conversion factors that can be written from this equality are

   \[
   \frac{1000 \text{ mg}}{\text{g}} \quad \text{and} \quad \frac{1 \text{ g}}{1000 \text{ mg}}
   \]
To derive an answer in mg, you’ll need to multiply 5.712 g by 1000 mg/g.

\[
5.712 \text{ g} \times \frac{1000 \text{ mg}}{\text{g}} = 5712 \text{ mg}
\]

The kilogram problem is solved similarly.

\[
1 \text{ kg} = 1000 \text{ g}
\]

Conversion factors representing this equality are

\[
\frac{1 \text{ kg}}{1000 \text{ g}} \quad \text{and} \quad \frac{1000 \text{ g}}{1 \text{ kg}}
\]

To derive an answer in kg, you’ll need to multiply 5.712 g by 1 kg/1000 g.

\[
5.712 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} = 0.005712 \text{ kg}
\]

The first answer makes sense because milligrams is a smaller unit than grams, and therefore there should be more milligrams. The second answer makes sense because kilograms is a larger unit than grams, and therefore there should be fewer kilograms.
The Greeks were among the many ancient peoples who sought to understand the nature of matter. One group of Greek philosophers, called the atomists, believed that matter could be broken down into pieces of a minute size. These pieces, called atoms or atomos, which means “indivisible,” possessed intrinsic, unchanging qualities. Another group of Greeks believed that matter could be divided an infinite number of times and could be changed from one type of matter into another.

Between 500 and 300 BCE, the Greek philosophers Leucippus and Democritus formulated the ideas that the atomists held. Leucippus and Democritus believed that all atoms were essentially the same but that the properties of all substances arose from the unique characteristics of their atoms. For example, solids, such as most metals, were thought to have uneven, jagged atoms. Because the atoms were rough, they could stick together and form solids. Similarly, water was thought to have atoms with smooth surfaces, which would allow the atoms to flow past one another. Though atomists did not have the same ideas about matter that we have today, they did believe that atoms were constantly in motion, even in objects that appeared to be solid.

Some Greek philosophers who studied matter between 700 and 300 BCE described matter in a way that differed from the way atomists described it. They attempted to identify and describe a fundamental substance from which all other matter was formed. Thales of Miletus (640–546 BCE) was among the first to suggest the existence of a basic element. He chose water, which exists as liquid, ice, and steam. He interpreted water’s changeability to mean that water could transform into any other substance. Other philosophers suggested that the basic element was air or fire. Empedokles (ca. 490–ca. 430 BCE) focused on four elements: earth, air, fire, and water. He thought that these elements combined in various proportions to make all known matter.

Aristotle (384–322 BCE), a student of Plato, elaborated on the earlier ideas about elements. He argued that in addition to the four elements that make up all matter, there were four basic properties: hot, cold, wet, and dry. In Aristotle’s view, the four elements could each have two of the basic properties. For example, water was wet and cold, while air was wet and hot. He thought that one element could change into another element if its properties were changed.

For more than 2,000 years, Aristotle’s classical ideas dominated scientific thought. It was not until the 1700s that the existence of atoms was shown experimentally and that the incredible intuition of the atomists was realized.

**Questions**

1. In Aristotle’s system of elements, fire opposes water. Why do you think Aristotle chose this relationship?
2. Use the ideas of the atomists to describe the atoms of the physical phases of matter—solid, liquid, and gas.
If you have ever measured something several times, you know that the results can vary. In science, for a reported measurement to be useful, there must be some indication of its reliability or uncertainty.

### MAIN IDEA

#### Accuracy is different from precision.

The terms **accuracy** and **precision** mean the same thing to most people. However, in science their meanings are quite distinct. **Accuracy** refers to the closeness of measurements to the correct or accepted value of the quantity measured. **Precision** refers to the closeness of a set of measurements of the same quantity made in the same way. Thus, measured values that are accurate are close to the accepted value. Measured values that are precise are close to one another but not necessarily close to the accepted value.

**Figure 3.1** on the facing page can help you visualize the difference between precision and accuracy. Several darts thrown separately at a dartboard may land in various positions, relative to the bull’s-eye and to one another. The closer the darts land to the bull’s-eye, the more accurately they were thrown. The closer they land to one another, the more precisely they were thrown. Thus, the set of results shown in **Figure 3.1a** is both accurate and precise: the darts are close to the bull’s-eye and close to each other. In **Figure 3.1b**, the set of results is inaccurate but precise: the darts are far from the bull’s-eye but close to each other. In **Figure 3.1c**, the set of results is both inaccurate and imprecise: the darts are far from the bull’s-eye and far from each other. Notice also that the darts are not evenly distributed around the bull’s-eye, so the set, even considered on average, is inaccurate. In **Figure 3.1d**, the set on average is accurate compared with the third case, but it is imprecise. That is because the darts are distributed evenly around the bull’s-eye but are far from each other.

#### Percentage Error

The accuracy of an individual value or of an average experimental value can be compared quantitatively with the correct or accepted value by calculating the percentage error. **Percentage error** is calculated by subtracting the accepted value from the experimental value, dividing the difference by the accepted value, and then multiplying by 100.
Percentage Error

Percentage error = \frac{\text{Value}_{\text{experimental}} - \text{Value}_{\text{accepted}}}{\text{Value}_{\text{accepted}}} \times 100

Percentage error has a negative value if the accepted value is greater than the experimental value. It has a positive value if the accepted value is less than the experimental value. The following sample problem illustrates the concept of percentage error.

Sample Problem C
A student measures the mass and volume of a substance and calculates its density as 1.40 g/mL. The correct, or accepted, value of the density is 1.30 g/mL. What is the percentage error of the student’s measurement?

\text{SOLVE}

\text{Percentage error} = \frac{\text{Value}_{\text{experimental}} - \text{Value}_{\text{accepted}}}{\text{Value}_{\text{accepted}}} \times 100

= \frac{1.40 \text{ g/mL} - 1.30 \text{ g/mL}}{1.30 \text{ g/mL}} \times 100 = 7.7\%

Practice

1. What is the percentage error for a mass measurement of 17.7 g, given that the correct value is 21.2 g?
2. A volume is measured experimentally as 4.26 mL. What is the percentage error, given that the correct value is 4.15 mL?
Error in Measurement

Some error or uncertainty always exists in any measurement. The skill of the measurer places limits on the reliability of results. The conditions of measurement also affect the outcome. The measuring instruments themselves place limitations on precision. Some balances can be read more precisely than others. The same is true of rulers, graduated cylinders, and other measuring devices.

When you use a properly calibrated measuring device, you can be almost certain of a particular number of digits in a reading. For example, you can tell that the nail in Figure 3.2 is definitely between 6.3 and 6.4 cm long. Looking more closely, you can see that the value is halfway between 6.3 and 6.4 cm. However, it is hard to tell whether the value should be read as 6.35 cm or 6.36 cm. The hundredths place is thus somewhat uncertain. Simply leaving it out would be misleading because you do have some indication of the value’s likely range. Therefore, you would estimate the value to the final questionable digit, perhaps reporting the length of the nail as 6.36 cm. You might include a plus-or-minus value to express the range, for example, 6.36 cm ± 0.01 cm.

Main idea

Significant figures are those measured precisely, plus one estimated digit.

In science, measured values are reported in terms of significant figures. Significant figures in a measurement consist of all the digits known with certainty plus one final digit, which is somewhat uncertain or is estimated. For example, in the reported nail length of 6.36 cm discussed above, the last digit, 6, is uncertain. All the digits, including the uncertain one, are significant, however. All contain information and are included in the reported value. Thus, the term significant does not mean certain. In any correctly reported measured value, the final digit is significant but not certain. Insignificant digits are never reported. As a chemistry student, you will need to use and recognize significant figures when you work with measured quantities and report your results, and when you evaluate measurements reported by others.

Determining the Number of Significant Figures

When you look at a measured quantity, you need to determine which digits are significant. That process is very easy if the number has no zeros, because all the digits shown are significant. For example, in a number reported as 3.95, all three digits are significant. The significance of zeros in a number depends on their location, however. You need to learn and follow several rules involving zeros. After you have studied the rules in Figure 3.3, use them to express the answers in the sample problem that follows.
### Rules for Determining Significant Zeros

<table>
<thead>
<tr>
<th>Rule</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Zeros appearing between nonzero digits are significant.</td>
<td>a. 40.7 L has three significant figures.</td>
</tr>
<tr>
<td></td>
<td>b. 87 009 km has five significant figures.</td>
</tr>
<tr>
<td>2. Zeros appearing in front of all nonzero digits are not significant.</td>
<td>a. 0.095 897 m has five significant figures.</td>
</tr>
<tr>
<td></td>
<td>b. 0.0009 kg has one significant figure.</td>
</tr>
<tr>
<td>3. Zeros at the end of a number and to the right of a decimal point are significant.</td>
<td>a. 85.00 g has four significant figures.</td>
</tr>
<tr>
<td></td>
<td>b. 9.000 000 000 mm has 10 significant figures.</td>
</tr>
<tr>
<td>4. Zeros at the end of a number but to the left of a decimal point may or may not be significant. If a zero has not been measured or estimated but is just a placeholder, it is not significant. A decimal point placed after zeros indicates that they are significant.</td>
<td>a. 2000 m may contain from one to four significant figures, depending on how many zeros are placeholders. <strong>For measurements given in this text, assume that 2000 m has one significant figure.</strong></td>
</tr>
<tr>
<td></td>
<td>b. 2000. m contains four significant figures, indicated by the presence of the decimal point.</td>
</tr>
</tbody>
</table>

---

**Sample Problem D** How many significant figures are in each of the following measurements?

a. 28.6 g  
b. 3440. cm  
c. 910 m  
d. 0.046 04 L  
e. 0.006 700 0 kg  

**Solve**  
Determine the number of significant figures in each measurement using the rules listed in Figure 3.3.

**a.** 28.6 g  
There are no zeros, so all three digits are significant.

**b.** 3440. cm  
By rule 4, the zero is significant because it is immediately followed by a decimal point; there are 4 significant figures.

**c.** 910 m  
By rule 4, the zero is not significant; there are 2 significant figures.

**d.** 0.046 04 L  
By rule 2, the first two zeros are not significant; by rule 1, the third zero is significant; there are 4 significant figures.

**e.** 0.006 700 0 kg  
By rule 2, the first three zeros are not significant; by rule 3, the last three zeros are significant; there are 5 significant figures.
When you perform calculations involving measurements, you need to know how to handle significant figures. This is especially true when you are using a calculator to carry out mathematical operations. The answers given on a calculator can be derived results with more digits than are justified by the measurements.

Suppose you used a calculator to divide a measured value of 154 g by a measured value of 327 mL. Each of these values has three significant figures. The calculator would show a numerical answer of 0.470948012. The answer contains digits not justified by the measurements used to calculate it. Such an answer has to be rounded off to make its degree of certainty match that in the original measurements. The answer should be 0.471 g/mL.

The rules for rounding are shown in Figure 3.4. The extent of rounding required in a given case depends on whether the numbers are being added, subtracted, multiplied, or divided.

### Figure 3.4

#### Rules for Rounding Numbers

<table>
<thead>
<tr>
<th>If the digit following the last digit to be retained is:</th>
<th>then the last digit should:</th>
<th>Example (rounded to three significant figures)</th>
</tr>
</thead>
<tbody>
<tr>
<td>greater than 5</td>
<td>be increased by 1</td>
<td>42.68 g ⟷ 42.7 g</td>
</tr>
<tr>
<td>less than 5</td>
<td>stay the same</td>
<td>17.32 m ⟷ 17.3 m</td>
</tr>
<tr>
<td>5, followed by nonzero digit(s)</td>
<td>be increased by 1</td>
<td>2.7851 cm ⟷ 2.79 cm</td>
</tr>
<tr>
<td>5, not followed by nonzero digit(s), and preceded by an odd digit</td>
<td>be increased by 1 (because 3 is odd)</td>
<td>4.635 kg ⟷ 4.64 kg</td>
</tr>
<tr>
<td>5, not followed by nonzero digit(s), and the preceding significant digit is even</td>
<td>stay the same (because 6 is even)</td>
<td>78.65 mL ⟷ 78.6 mL</td>
</tr>
</tbody>
</table>
Addition or Subtraction with Significant Figures

Consider two mass measurements, 25.1 g and 2.03 g. The first measurement, 25.1 g, has one digit to the right of the decimal point, in the tenths place. There is no information on possible values for the hundredths place. That place is simply blank and cannot be assumed to be zero. The other measurement, 2.03 g, has two digits to the right of the decimal point. It provides information up to and including the hundredths place.

Suppose you were asked to add the two measurements. Simply carrying out the addition would result in an answer of 25.1 g + 2.03 g = 27.13 g. That answer suggests there is certainty all the way to the hundredths place. However, that result is not justified because the hundredths place in 25.1 g is completely unknown. The answer must be adjusted to reflect the uncertainty in the numbers added.

When adding or subtracting decimals, the answer must have the same number of digits to the right of the decimal point as there are in the measurement having the fewest digits to the right of the decimal point. When you compare the two values 25.1 g and 2.03 g, the measurement with the fewest digits to the right of the decimal point is 25.1 g. It has only one such digit. Following the rule, the answer must be rounded so that it has no more than one digit to the right of the decimal point. The answer should therefore be rounded to 27.1 g.

When working with whole numbers, the answer should be rounded so that the final significant digit is in the same place as the leftmost uncertain digit. For example, 5400 + 365 = 5800.

Multiplication and Division with Significant Figures

Suppose you calculated the density of an object that has a mass of 3.05 g and a volume of 8.47 mL. The following division on a calculator will give a value of 0.360094451.

\[
\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{3.05 \text{ g}}{8.47 \text{ mL}} = 0.360094451 \text{ g/mL}
\]

The answer must be rounded to the correct number of significant figures. The values of mass and volume used to obtain the answer have only three significant figures each. The degree of certainty in the calculated result is not justified. For multiplication or division, the answer can have no more significant figures than are in the measurement with the fewest number of significant figures. In the calculation just described, the answer, 0.360094451 g/mL, would be rounded to three significant figures to match the significant figures in 8.47 mL and 3.05 g. The answer would thus be 0.360 g/mL.

CHECK FOR UNDERSTANDING

Analyze Suppose you measure the classroom once using a piece of rope you know to be 10 m long and again with a measuring tape marked in m, cm, and mm. You then take the average of the two measurements. Which would determine the number of significant figures in your answer? Explain your answer.
Conversion Factors and Significant Figures

Earlier in this chapter, you learned how conversion factors are used to change one unit to another. Such conversion factors are typically exact. That is, there is no uncertainty in them. For example, there are exactly 100 cm in a meter. If you were to use the conversion factor 100 cm/m to change meters to centimeters, the 100 would not limit the degree of certainty in the answer. Thus, 4.608 m could be converted to centimeters as follows.

\[ 4.608 \text{ m} \times \frac{100 \text{ cm}}{\text{m}} = 460.8 \text{ cm} \]

The answer still has four significant figures. Because the conversion factor is considered exact, the answer would not be rounded. Most exact conversion factors are defined, rather than measured, quantities.

Counted numbers also produce conversion factors of unlimited precision. For example, if you counted that there are 10 test tubes for every student, that would produce an exact conversion factor of 10 test tubes/student. There is no uncertainty in that factor.
Scientific notation is used to express very large or very small numbers.

In scientific notation, numbers are written in the form \( M \times 10^n \), where the factor \( M \) is a number greater than or equal to 1 but less than 10, and \( n \) is a whole number. For example, to put the quantity 65 000 km in scientific notation and show the first two digits as significant, you would write:

\[
6.5 \times 10^4 \text{ km}
\]

Writing the \( M \) factor as 6.5 shows that there are exactly two significant figures. If, instead, you intended the first three digits in 65 000 to be significant, you would write \( 6.50 \times 10^4 \text{ km} \). When numbers are written in scientific notation, only the significant figures are shown.

Suppose you are expressing a very small quantity, such as the length of a flu virus. In ordinary notation this length could be 0.000 12 mm. That length can be expressed in scientific notation as follows.

\[
0.000 \text{ mm} = 1.2 \times 10^{-4} \text{ mm}
\]

Move the decimal point four places to the right, and multiply the number by \( 10^{-4} \).

1. **Determine** \( M \) by moving the decimal point in the original number to the left or the right so that only one nonzero digit remains to the left of the decimal point.

2. **Determine** \( n \) by counting the number of places that you moved the decimal point. If you moved it to the left, \( n \) is positive. If you moved it to the right, \( n \) is negative.

### Mathematical Operations Using Scientific Notation

1. **Addition and subtraction** These operations can be performed only if the values have the same exponent (\( n \) factor). If they do not, adjustments must be made to the values so that the exponents are equal. Once the exponents are equal, the \( M \) factors can be added or subtracted. The exponent of the answer can remain the same, or it may then require adjustment if the \( M \) factor of the answer has more than one digit to the left of the decimal point. Consider the example of the addition of \( 4.2 \times 10^4 \text{ kg} \) and \( 7.9 \times 10^3 \text{ kg} \). We can make both exponents either 3 or 4. The following solutions are possible.

\[
\begin{align*}
4.2 \times 10^4 \text{ kg} \\
+ 0.79 \times 10^4 \text{ kg} \\
4.99 \times 10^4 \text{ kg} \text{ rounded to } 5.0 \times 10^4 \text{ kg}
\end{align*}
\]

or

\[
\begin{align*}
7.9 \times 10^3 \text{ kg} \\
+ 42 \times 10^3 \text{ kg} \\
49.9 \times 10^3 \text{ kg} = 4.99 \times 10^4 \text{ kg} \text{ rounded to } 5.0 \times 10^4 \text{ kg}
\end{align*}
\]

Note that the units remain kg throughout.
2. Multiplication The $M$ factors are multiplied, and the exponents are added algebraically.

Consider the multiplication of $5.23 \times 10^6 \mu m$ by $7.1 \times 10^{-2} \mu m$.

$$(5.23 \times 10^6 \mu m)(7.1 \times 10^{-2} \mu m) = (5.23 \times 7.1)(10^6 \times 10^{-2})$$

$$= 37.133 \times 10^4 \mu m^2 \text{ (adjust to two significant digits)}$$

$$= 3.7 \times 10^5 \mu m^2$$

Note that when length measurements are multiplied, the result is area. The unit is now $\mu m^2$.

3. Division The $M$ factors are divided, and the exponent of the denominator is subtracted from that of the numerator. The calculator keystrokes for this problem are shown in Figure 3.5.

$$\frac{5.44 \times 10^7 g}{8.1 \times 10^4 \text{ mol}} = \frac{5.44}{8.1} \times 10^{7-4} \text{ g/mol}$$

$$= 0.6716049383 \times 10^3 \text{ (adjust to two significant digits)}$$

$$= 6.7 \times 10^2 \text{ g/mol}$$

Note that the unit for the answer is the ratio of grams to moles.

**Sample problems are guides to solving similar types of problems.**

Learning to analyze and solve such problems requires practice and a logical approach. In this section, you will review a process that can help you analyze problems effectively. Most sample problems in this book are organized by four basic steps to guide your thinking in how to work out the solution to a problem.
Step 1. Analyze

The first step in solving a quantitative word problem is to read the problem carefully at least twice and to analyze the information in it. Note any important descriptive terms that clarify or add meaning to the problem. Identify and list the data given in the problem. Also identify the unknown—the quantity you are asked to find.

Step 2. Plan

The second step is to develop a plan for solving the problem. The plan should show how the information given is to be used to find the unknown. In the process, reread the problem to make sure you have gathered all the necessary information. It is often helpful to draw a picture that represents the problem. For example, if you were asked to determine the volume of a crystal given its dimensions, you could draw a representation of the crystal and label the dimensions. This drawing would help you visualize the problem.

Decide which conversion factors, mathematical formulas, or chemical principles you will need to solve the problem. Your plan might suggest a single calculation or a series of them involving different conversion factors. Once you understand how you need to proceed, you may wish to sketch out the route you will take, using arrows to point the way from one stage of the solution to the next. Sometimes you will need data that are not actually part of the problem statement. For instance, you’ll often use data from the periodic table.

Step 3. Solve

The third step involves substituting the data and necessary conversion factors into the plan you have developed. At this stage you calculate the answer, cancel units, and round the result to the correct number of significant figures. It is very important to have a plan worked out in step 2 before you start using the calculator. All too often, students start multiplying or dividing values given in the problem before they really understand what they need to do to get an answer.

Step 4. Check Your Work

Examine your answer to determine whether it is reasonable. Use the following methods, when appropriate, to carry out the evaluation.

1. Check to see that the units are correct. If they are not, look over the setup. Are the conversion factors correct?

2. Make an estimate of the expected answer. Use simpler, rounded numbers to do so. Compare the estimate with your actual result. The two should be similar.

3. Check the order of magnitude in your answer. Does it seem reasonable compared with the values given in the problem? If you calculated the density of vegetable oil and got a value of 54.9 g/mL, you should know that something is wrong. Oil floats on water.
Therefore, its density is less than water. So, the value obtained should be less than 1.0 g/mL.

4. Be sure that the answer given for any problem is expressed using the correct number of significant figures.

Look over the following quantitative Sample Problem. Notice how the four-step approach is used, and then apply the approach yourself in solving the practice problems that follow.

### Solving Problems Using the Four-Step Approach

#### Sample Problem F

Calculate the volume of a sample of aluminum that has a mass of 3.057 kg. The density of aluminum is 2.70 g/cm³.

1. **ANALYZE**

   **Given:**
   - mass = 3.057 kg
   - density = 2.70 g/cm³

   **Unknown:**
   - volume of aluminum

2. **PLAN**

   The density unit in the problem is g/cm³, and the mass given in the problem is expressed in kg. Therefore, in addition to using the density equation, you will need a conversion factor representing the relationship between grams and kilograms.

   \[
   1000 \text{ g} = 1 \text{ kg}
   \]

   Also, rearrange the density equation to solve for volume.

   \[
   \text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{or} \quad D = \frac{m}{V}
   \]

   \[
   V = \frac{m}{D}
   \]

3. **SOLVE**

   \[
   V = \frac{3.057 \text{ kg}}{2.70 \text{ g/cm}^3} \times \frac{1000 \text{ g}}{1 \text{ kg}} = 1132.222 \ldots \text{ cm}^3 \quad \text{(calculator answer)}
   \]

   The answer should be rounded to three significant figures.

   \[
   V = 1.13 \times 10^3 \text{ cm}^3
   \]

4. **CHECK YOUR WORK**

   The unit of volume, cm³, is correct. An order-of-magnitude estimate would put the answer at over 1000 cm³.

   \[
   \frac{3}{2} \times 1000
   \]

   The correct number of significant figures is three, which matches that in 2.70 g/cm³.

### Practice

**Answers in Appendix E**

1. What is the volume, in milliliters, of a sample of helium that has a mass of \(1.73 \times 10^{-3}\) g, given that the density is 0.17847 g/L?

2. What is the density of a piece of metal that has a mass of \(6.25 \times 10^5\) g and is \(92.5\) cm \(\times\) \(47.3\) cm \(\times\) \(85.4\) cm?

3. How many millimeters are there in \(5.12 \times 10^5\) kilometers?

4. A clock gains 0.020 second per minute. How many seconds will the clock gain in exactly six months, assuming exactly 30 days per month?
**MAIN IDEA**

Variables that are directly proportional increase or decrease by the same factor.

Two quantities are directly proportional to each other if dividing one by the other gives a constant value. For example, if the masses and volumes of different samples of aluminum are measured, the masses and volumes will be directly proportional to each other. As the masses of the samples increase, their volumes increase by the same factor, as you can see from the data. Doubling the mass doubles the volume. Halving the mass halves the volume.

When two variables, \( x \) and \( y \), are directly proportional to each other, the relationship can be expressed as \( y \propto x \), which is read as “\( y \) is proportional to \( x \).” The general equation for a directly proportional relationship between the two variables can also be written as follows.

\[
\frac{y}{x} = k
\]

The value of \( k \) is a constant called the proportionality constant. Written in this form, the equation expresses an important fact about direct proportion: the ratio between the variables remains constant. Note that using the mass and volume values in Figure 3.6 gives a mass-volume ratio that is constant (neglecting measurement error). The equation can be rearranged into the following form.

\[
y = kx
\]

The equation \( y = kx \) may look familiar to you. It is the equation for a special case of a straight line. If two variables related in this way are graphed versus one another, a straight line, or linear plot that passes through the origin, results. The data for aluminum from Figure 3.6 are graphed in Figure 3.7. The mass and volume of a pure substance are directly proportional to each other. Consider mass to be \( y \) and volume to be \( x \). The constant ratio, \( k \), for the two variables is density. The slope of the line reflects the constant density, or mass-volume ratio.

**Figure 3.6**

**MASS-VOLUME DATA FOR ALUMINUM AT 20°C**

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Volume (cm³)</th>
<th>( \frac{m}{V} ) (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>54.7</td>
<td>20.1</td>
<td>2.72</td>
</tr>
<tr>
<td>65.7</td>
<td>24.4</td>
<td>2.69</td>
</tr>
<tr>
<td>83.5</td>
<td>30.9</td>
<td>2.70</td>
</tr>
<tr>
<td>96.3</td>
<td>35.8</td>
<td>2.69</td>
</tr>
<tr>
<td>105.7</td>
<td>39.1</td>
<td>2.70</td>
</tr>
</tbody>
</table>

**Figure 3.7**

**Mass vs. Volume** The graph of mass versus volume shows a relationship of direct proportion. Notice that the line is extrapolated to pass through the origin.

**Measurements and Calculations**
For aluminum, this value is 2.70 g/cm³ at 20°C. Notice also that the plotted line passes through the origin (0,0). All directly proportional relationships produce linear graphs that pass through the origin.

**MAIN IDEA**

**Quantities are inversely proportional if one decreases in value when the other increases.**

Two quantities are **inversely proportional** to each other if their product is constant. An example of an inversely proportional relationship is that between speed of travel and the time required to cover a fixed distance. The greater the speed, the less time that is needed to go a certain fixed distance. Doubling the speed cuts the required time in half. Halving the speed doubles the required time.

When two variables, \(x\) and \(y\), are inversely proportional to each other, the relationship can be expressed as follows.

\[ y \propto \frac{1}{x} \]

This is read “\(y\) is proportional to 1 divided by \(x\).” The general equation for an inversely proportional relationship between the two variables can be written in the following form.

\[ xy = k \]

In the equation, \(k\) is the proportionality constant. If \(x\) increases, \(y\) must decrease by the same factor to keep the product constant.

When the temperature of a sample of nitrogen is kept constant, the volume \((V)\) of the gas sample decreases as the pressure \((P)\) increases, as shown in **Figure 3.8**. Note that \(P \times V\) gives a reasonably constant value. Thus, \(P\) and \(V\) are inversely proportional to each other. The graph of this data is shown in **Figure 3.9**. A graph of variables that are inversely proportional produces a curve called a **hyperbola**.

### Figure 3.8

**PRESSURE-VOLUME DATA FOR NITROGEN AT CONSTANT TEMPERATURE**

<table>
<thead>
<tr>
<th>Pressure (kPa)</th>
<th>Volume (cm³)</th>
<th>(P \times V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>500</td>
<td>50 000</td>
</tr>
<tr>
<td>150</td>
<td>333</td>
<td>50 000</td>
</tr>
<tr>
<td>200</td>
<td>250</td>
<td>50 000</td>
</tr>
<tr>
<td>250</td>
<td>200</td>
<td>50 000</td>
</tr>
<tr>
<td>300</td>
<td>166</td>
<td>49 800</td>
</tr>
<tr>
<td>350</td>
<td>143</td>
<td>50 100</td>
</tr>
<tr>
<td>400</td>
<td>125</td>
<td>50 000</td>
</tr>
<tr>
<td>450</td>
<td>110</td>
<td>49 500</td>
</tr>
</tbody>
</table>
Reviewing Main Ideas

1. The density of copper is listed as 8.94 g/cm³. Two students each make three density determinations of samples of the substance. Student A’s results are 7.3 g/mL, 9.4 g/mL, and 8.3 g/mL. Student B’s results are 8.4 g/cm³, 8.8 g/cm³, and 8.0 g/cm³. Compare the two sets of results in terms of precision and accuracy.

2. Determine the number of significant figures.
   a. 6.002 cm
   b. 0.0020 m
   c. 10.0500 g
   d. 7000 kg
   e. 7000. kg

3. Round 2.6765 to two significant figures.

4. Carry out the following calculations.
   a. 52.13 g + 1.7502 g
   b. 12 m × 6.41 m
   c. \( \frac{16.25}{5.1442} \) mL

5. Perform the following operations. Express each answer in scientific notation.
   a. \( (1.54 \times 10^{-2} \text{ g}) + (2.86 \times 10^{-1} \text{ g}) \)
   b. \( (7.023 \times 10^9 \text{ g}) - (6.62 \times 10^7 \text{ g}) \)
   c. \( (8.99 \times 10^{-4} \text{ m}) \times (3.57 \times 10^4 \text{ m}) \)
   d. \( \frac{2.17 \times 10^{-3} \text{ g}}{5.002 \times 10^4 \text{ mL}} \)

6. Write the following numbers in scientific notation.
   a. 560 000
   b. 33 400
   c. 0.000 4120

7. A student measures the mass of a beaker filled with corn oil. The mass reading averages 215.6 g. The mass of the beaker is 110.4 g.
   a. What is the mass of the corn oil?
   b. What is the density of the corn oil if its volume is 114 cm³?

8. Calculate the mass of gold that occupies \( 5.0 \times 10^{-3} \text{ cm}^3 \). The density of gold is 19.3 g/cm³.

9. What is the difference between a graph representing data that are directly proportional and a graph of data that are inversely proportional?

Critical Thinking

10. APPLYING CONCEPTS The mass of a liquid is 11.50 g, and its volume is 9.03 mL. How many significant figures should its density value have? Explain the reason for your answer.
Any value expressed in scientific notation, whether large or small, has two parts. The first part, the *first factor*, consists of a number greater than or equal to 1 but less than 10. It may have any number of digits after the decimal point. The second part consists of a power of 10.

\[ 6.02 \times 10^{23} \]

To write the first part, move the decimal point to the right or the left so that there is only one nonzero digit to the left of the decimal point. The second part is written as an exponent, which is determined by counting the number of places the decimal point must be moved. If it is moved to the right, the exponent is negative. If it is moved to the left, the exponent is positive.

### Problem-Solving TIPS

- In addition and subtraction, all values must first be converted to numbers that have the same exponent of 10. The result is the sum or the difference of the first factors, multiplied by the same exponent of 10. Finally, the result should be rounded to the correct number of significant figures and expressed in scientific notation.
- In multiplication, the first factors are multiplied and the exponents of 10 are added.
- In division, the *first factors* of the numbers are divided, and the exponent of 10 in the denominator is subtracted from the exponent of 10 in the numerator.

### Sample Problems

**Write 299 800 000 m/s in scientific notation.**

The decimal must move to the left 8 places, which indicates a positive exponent.

\[ 299 \, 800 \, 000 \, \text{m/s} \]

The value in scientific notation is \( 2.998 \times 10^8 \) m/s.

**Solve the following equation and write the answer in scientific notation.**

\((3.1 \times 10^3)(5.21 \times 10^4)\)

Multiply the first factors, and then add the exponents of 10.

\[(3.1 \times 5.21) \times 10^{(3+4)} = 16 \times 10^7 = 1.6 \times 10^8\]

### Practice

1. Rewrite the following numbers in scientific notation.
   - a. 0.0000745 g
   - b. 5984102 nm

2. Solve the following equations, and write the answers in scientific notation.
   - a. \( 1.017 \times 10^3 - 1.013 \times 10^4 \)
   - b. \( 9.27 \times 10^4 \div 11.24 \times 10^5 \)
### SECTION 1 Scientific Method

- The scientific method is a logical approach to solving problems that lend themselves to investigation.
- A hypothesis is a testable statement that serves as the basis for predictions and further experiments.
- A theory is a broad generalization that explains a body of known facts or phenomena.

<table>
<thead>
<tr>
<th>KEY TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>scientific method</td>
</tr>
<tr>
<td>system</td>
</tr>
<tr>
<td>hypothesis</td>
</tr>
<tr>
<td>model</td>
</tr>
<tr>
<td>theory</td>
</tr>
</tbody>
</table>

### SECTION 2 Units of Measurement

- The result of nearly every measurement is a number and a unit.
- The SI system of measurement is used in science. It has seven base units: the meter (length), kilogram (mass), second (time), kelvin (temperature), mole (amount of substance), ampere (electric current), and candela (luminous intensity).
- Weight is a measure of the gravitational pull on matter.
- Derived SI units include the square meter (area) and the cubic meter (volume).
- Density is the ratio of mass to volume.
- Conversion factors are used to convert from one unit to another.

<table>
<thead>
<tr>
<th>KEY TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>quantity</td>
</tr>
<tr>
<td>SI</td>
</tr>
<tr>
<td>weight</td>
</tr>
<tr>
<td>derived unit</td>
</tr>
<tr>
<td>volume</td>
</tr>
<tr>
<td>density</td>
</tr>
<tr>
<td>conversion factor</td>
</tr>
<tr>
<td>dimensional analysis</td>
</tr>
</tbody>
</table>

### SECTION 3 Using Scientific Measurements

- Accuracy refers to the closeness of a measurement to the correct or accepted value. Precision refers to the closeness of values for a set of measurements.
- Percentage error is the difference between the experimental and the accepted value that is divided by the accepted value and then multiplied by 100.
- The significant figures in a number consist of all digits known with certainty plus one final digit, which is uncertain.
- After addition or subtraction, the answer should be rounded so that it has no more digits to the right of the decimal point than there are in the measurement that has the smallest number of digits to the right of the decimal point. After multiplication or division, the answer should be rounded so that it has no more significant figures than there are in the measurement that has the fewest number of significant figures.
- Exact conversion factors are completely certain and do not limit the number of digits in a calculation.
- A number written in scientific notation is of the form $M \times 10^n$, in which $M$ is greater than or equal to 1 but less than 10, and $n$ is an integer.
- Two quantities are directly proportional to each other if dividing one by the other yields a constant value. Two quantities are inversely proportional to each other if their product has a constant value.

<table>
<thead>
<tr>
<th>KEY TERMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>accuracy</td>
</tr>
<tr>
<td>precision</td>
</tr>
<tr>
<td>percentage error</td>
</tr>
<tr>
<td>significant figures</td>
</tr>
<tr>
<td>scientific notation</td>
</tr>
<tr>
<td>directly proportional</td>
</tr>
<tr>
<td>inversely proportional</td>
</tr>
</tbody>
</table>
SECTION 1
Scientific Method

REVIEWING MAIN IDEAS
1. How does quantitative information differ from qualitative information?
2. What is a hypothesis?
3. a. What is a model in the scientific sense?
   b. How does a model differ from a theory?

SECTION 2
Units of Measurement

REVIEWING MAIN IDEAS
4. Why is it important for a measurement system to have an international standard?
5. How does a quantity differ from a unit? Use two examples to explain the difference.
6. List the seven SI base units and the quantities they represent.
7. What is the numerical equivalent of each of the following SI prefixes?
   a. kilo-
   b. centi-
   c. micro-
   d. milli-
   e. mega-
8. Identify the SI unit that would be most appropriate for expressing the length of the following.
   a. width of a gymnasium
   b. length of a finger
   c. distance between your town and the closest border of the next state
   d. length of a bacterial cell
9. Identify the SI unit that would be most appropriate for measuring the mass of each of the following objects.
   a. table
   b. coin
   c. a 250 mL beaker
10. Explain why the second is not defined by the length of the day.
11. a. What is a derived unit?
    b. What is the SI-derived unit for area?
12. a. List two SI-derived units for volume.
    b. List two non-SI units for volume, and explain how they relate to the cubic centimeter.
13. a. Why are the units that are used to express the densities of gases different from those used to express the densities of solids or liquids?
    b. Name two units for density.
    c. Why is the temperature at which density is measured usually specified?
14. a. Which of the solids listed in Figure 2.8 will float on water?
    b. Which of the liquids will sink in milk?
15. a. Define conversion factor.
    b. Explain how conversion factors are used.

PRACTICE PROBLEMS
16. What is the volume, in cubic meters, of a rectangular solid that is 0.25 m long, 6.1 m wide, and 4.9 m high?
17. Find the density of a material, given that a 5.03 g sample occupies 3.24 mL. (Hint: See Sample Problem A.)
18. What is the mass of a sample of material that has a volume of 55.1 cm³ and a density of 6.72 g/cm³?
19. A sample of a substance that has a density of 0.824 g/mL has a mass of 0.451 g. Calculate the volume of the sample.
20. How many grams are in 882 µg? (Hint: See Sample Problem B.)
21. Calculate the number of milliliters in 0.603 L.
22. The density of gold is 19.3 g/cm³.
   a. What is the volume, in cubic centimeters, of a sample of gold that has a mass of 0.715 kg?
   b. If this sample of gold is a cube, what is the length of each edge in centimeters?
23. a. Find the number of kilometers in 92.25 m.
    b. Convert the answer in kilometers to centimeters.
SECTION 3
Using Scientific Measurements

REVIEWING MAIN IDEAS

24. Compare accuracy and precision.

25. a. Write the equation that is used to calculate percentage error.
   b. Under what condition will percentage error be negative?

26. How is the average for a set of values calculated?

27. What is meant by a mass measurement expressed in this form: 4.6 g ± 0.2 g?

28. Suppose a graduated cylinder were not correctly calibrated. How would this affect the results of a measurement? How would it affect the results of a calculation using this measurement?

29. Round each of the following measurements to the number of significant figures indicated.
   a. 67.029 g to three significant figures
   b. 0.15 L to one significant figure
   c. 52.8005 mg to five significant figures
   d. 3.174 97 mol to three significant figures

30. State the rules governing the number of significant figures that result from each of the following operations.
   a. addition and subtraction
   b. multiplication and division

31. What is the general form for writing numbers in scientific notation?

32. a. By using x and y, state the general equation for quantities that are directly proportional.
   b. For two directly proportional quantities, what happens to one variable when the other variable increases?

33. a. State the general equation for quantities, x and y, that are inversely proportional.
   b. For two inversely proportional quantities, what happens to one variable when the other increases?

34. Arrange in the correct order the following four basic steps for finding the solution to a problem: check your work, analyze, solve, and plan.

PRACTICE PROBLEMS

35. A student measures the mass of a sample as 9.67 g. Calculate the percentage error, given that the correct mass is 9.82 g. (Hint: See Sample Problem C.)

36. A handbook gives the density of calcium as 1.54 g/cm³. Based on lab measurements, what is the percentage error of a density calculation of 1.25 g/cm³?

37. What is the percentage error of a length measurement of 0.229 cm if the correct value is 0.225 cm?

38. How many significant figures are in each of the following measurements? (Hint: See Sample Problem D.)
   a. 0.4004 mL
   b. 6000 g
   c. 1.000 30 km
   d. 400. mm

39. Calculate the sum of 6.078 g and 0.3329 g.

40. Subtract 7.11 cm from 8.2 cm. (Hint: See Sample Problem E.)

41. What is the product of 0.8102 m and 3.44 m?

42. Divide 94.20 g by 3.167 22 mL.

43. Write the following numbers in scientific notation.
   a. 0.000 673 0
   b. 50 000.0
   c. 0.000 003 010

44. The following numbers are in scientific notation. Write them in ordinary notation.
   a. 7.050 × 10³ g
   b. 4.000 05 × 10⁻² mg
   c. 2.350 0 × 10⁻¹ mL

45. Perform the following operation. Express the answer in scientific notation and with the correct number of significant figures.
   0.002115 m × 0.0000405 m

46. A sample of a certain material has a mass of 2.03 × 10⁻³ g. Calculate the volume of the sample, given that the density is 9.133 × 10⁻¹ g/cm³. Use the four-step method to solve the problem. (Hint: See Sample Problem F.)

Chapter Review
Mixed Review

47. A man finds that he has a mass of 100.6 kg. He goes on a diet, and several months later he finds that he has a mass of 96.4 kg. Express each number in scientific notation, and calculate the number of kilograms the man has lost by dieting.

48. A large office building is $1.07 \times 10^2$ m long, 31 m wide, and $4.25 \times 10^2$ m high. What is its volume?

49. An object has a mass of 57.6 g. Find the object’s density, given that its volume is 40.25 cm$^3$.

50. A lab worker measures the mass of some sucrose as 0.947 mg. Convert that quantity to grams and to kilograms.

51. A student calculates the density of iron as 6.80 g/cm$^3$ by using lab data for mass and volume. A handbook reveals that the correct value is 7.86 g/cm$^3$. What is the percentage error?

52. Find the table of properties for Group 1 elements in the Elements Handbook (Appendix A). Calculate the volume of a single atom of each element listed in the table by using the equation for the volume of a sphere.

$$V = \frac{4}{3} \pi r^3$$

53. Use the radius of a sodium atom from the Elements Handbook (Appendix A) to calculate the number of sodium atoms in a row 5.00 cm long. Assume that each sodium atom touches the ones next to it.

54. a. A block of sodium that has the measurements 3.00 cm $\times$ 5.00 cm $\times$ 5.00 cm has a mass of 75.5 g. Calculate the density of sodium.

b. Compare your calculated density with the value in the properties table for Group 1 elements. Calculate the percentage error for your density determination.

55. How does the metric system, which was once a standard for measurement, differ from SI? Why was it necessary for the United States to change to SI?

56. What are ISO 9000 standards? How do they affect industry on an international level?

57. Performance Obtain three metal samples from your teacher. Determine the mass and volume of each sample. Calculate the density of each metal from your measurement data. (Hint: Consider using the water displacement technique to measure the volume of your samples.)

58. Use the data from the Nutrition Facts label below to answer the following questions:

a. Use the data given on the label for grams of fat and calories from fat to construct a conversion factor that has the units calories per gram.

b. Calculate the mass in kilograms for 20 servings of the food.

c. Calculate the mass of protein in micrograms for one serving of the food.

d. What is the correct number of significant figures for the answer in item a? Why?

### Nutrition Facts

<table>
<thead>
<tr>
<th>Nutrition Facts Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Serving Size: 1/4 cup (30g)</td>
</tr>
<tr>
<td>Servings Per Container: About 14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Amount Per Serving</th>
<th>Calories: 120</th>
<th>Calories from Fat: 15</th>
</tr>
</thead>
<tbody>
<tr>
<td>with Corn Crunch</td>
<td>160</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>% Daily Value**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Fat 2g*</td>
</tr>
<tr>
<td>Saturated Fat 0g</td>
</tr>
<tr>
<td>Cholesterol 0mg</td>
</tr>
<tr>
<td>Sodium 160mg</td>
</tr>
<tr>
<td>Potassium 65mg</td>
</tr>
</tbody>
</table>

| Total Carbohydrate: 25g | 8% |
| Dietary Fiber: 3g       |    |
| Sugars: 3g               |    |
| Other Carbohydrate: 11g  |    |

| Protein: 2g |

*Amount in Cereal. A serving of cereal plus skim milk provides 2g fat, less 5mg cholesterol, 220mg sodium, 270mg potassium, 31g carbohydrate (11g sugars) and 6g protein.

**Percent Daily Values are based on a 2,000 calorie diet. Your daily values may be higher or lower depending on your calorie needs.

<table>
<thead>
<tr>
<th>Calories</th>
<th>Total Carbohydrate</th>
<th>Dietary Fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>2,000</td>
<td>300g</td>
<td>25g</td>
</tr>
<tr>
<td>2,500</td>
<td>350g</td>
<td>25g</td>
</tr>
<tr>
<td>Less than 65g</td>
<td>Less than 20g</td>
<td>Less than 300g</td>
</tr>
</tbody>
</table>
Standards-Based Assessment

Answer the following items on a separate piece of paper.

MULTIPLE CHOICE

1. Which of the following masses is the largest?
   A. 0.200 g
   B. 0.020 kg
   C. 20.0 mg
   D. 2000 μg

2. Which of the following measurements contains three significant figures?
   A. 200 mL
   B. 0.02 mL
   C. 20.2 mL
   D. 200.0 mL

3. A theory differs from a hypothesis in that a theory
   A. cannot be disproved.
   B. always leads to the formation of a law.
   C. has been subjected to experimental testing.
   D. represents an educated guess.

4. All measurements in science
   A. must be expressed in scientific notation.
   B. have some degree of uncertainty.
   C. are both accurate and precise.
   D. must include only those digits that are known with certainty.

5. When numbers are multiplied or divided, the answer can have no more
   A. significant figures than are in the measurement that has the smallest number of significant figures.
   B. significant figures than are in the measurement that has the largest number of significant figures.
   C. digits to the right of the decimal point than are in the measurement that has the smallest number of digits to the right of the decimal point.
   D. digits to the right of the decimal point than are in the measurement that has the largest number of digits to the right of the decimal point.

6. Which of the following is not part of the scientific method?
   A. making measurements
   B. introducing bias
   C. making an educated guess
   D. analyzing data

7. The accuracy of a measurement
   A. is how close it is to the true value.
   B. does not depend on the instrument used to measure the object.
   C. indicates that the measurement is also precise.
   D. is something that scientists rarely achieve.

8. A measurement of 23 465 mg converted to grams equals
   A. 2.3465 g.
   B. 23.465 g.
   C. 234.65 g.
   D. 0.23465 g.

9. A metal sample has a mass of 43.65 g. The volume of the sample is 16.9 cm³. The density of the sample is
   A. 2.7 g/cm³.
   B. 2.70 g/cm³.
   C. 0.370 g/cm³.
   D. 0.37 g/cm³.

SHORT ANSWER

10. A recipe for 18 cookies calls for 1 cup of chocolate chips. How many cups of chocolate chips are needed for 3 dozen cookies? What kind of proportion, direct or indirect, did you use to answer this question?

11. Which of the following statements contain exact numbers?
   A. There are 12 eggs in a dozen.
   B. The accident injured 21 people.
   C. The circumference of the Earth at the equator is 40,000 km.

EXTENDED RESPONSE

12. You have decided to test the effects of five garden fertilizers by applying some of each to separate rows of radishes. What is the variable you are testing? What factors should you control? How will you measure and analyze the results?

13. Around 1150, King David I of Scotland defined the inch as the width of a man’s thumb at the base of the nail. Discuss the practical limitations of this early unit of measurement.